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On vortex strength and drag in bluff-body wakes

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In a recent paper (Griffin & Ramberg 1974) the authors studied the vortex-street wakes behind forced vibrating rigid cylinders. All experimental conditions were within the regime of wake capture or synchronization between the vibration and vortex frequencies. Both mean and fluctuating velocities in the wake together with the length of the vortex formation region were measured as functions of vibration amplitude and frequency at a Reynolds number of 144. The viscous vortex strength, age and spacing at this Reynolds number were then obtained by matching a model for the vortex street with the mean and r.m.s. velocity profiles obtained from hot-wire measurements. These results are employed here to determine the steady drag force on the vibrating cylinder by means of the von Kármán drag formulation. The drag coefficients determined in this way are in agreement with the recently published direct force measurements of Tanida, Okajima & Watanabe (1973) and Griffin, Skop & Koopmann (1973) at Reynolds numbers of 80, 500–900 and 4000. From these results a direct relation is drawn between the increased drag on resonantly vibrating structures and changes in the vortex strength, spacing and formation in their wakes.

1. Introduction

If one of the natural frequencies of a bluff body immersed in a moving stream of fluid is near the frequency at which vortices are naturally shed from the body, then self-excited resonant vibrations can occur if the damping of the system is sufficiently low. There is also a range of frequencies near this so-called Strouhal frequency of vortex shedding where forced vibrations of the body cause the vortex shedding frequency to be captured by, or to synchronize with, the body frequency. This means that the body and wake have the same characteristic frequency and that the Strouhal frequency, relating to the vortex shedding from a stationary body, is suppressed. The forces which act on a structure are amplified as a result of such vibrations, and these forces are closely related to the changes which occur in the wake flow downstream of the body.

The purpose of this paper is to examine further some results recently published by the authors (see Griffin & Ramberg 1974). In that paper the vortex strength and spacing in the wake of a vibrating cylinder were obtained both by means of flow visualization and by matching measured wake velocity profiles with a model for the vortex street, and the results obtained by the two experimental methods were in good agreement. These hydrodynamic properties of the bluff-body wake are employed in the present paper to determine the drag on a vibrating cylinder. The values of the drag coefficient obtained from wake properties by means of the von Kármán drag formula are in agreement with direct force measurements which have only recently become available.

2. The von Kármán drag formulation

The steady drag force on a bluff body in a uniform stream can be computed by means of the drag formulation which was derived by von Kármán, and which is discussed in detail by Milne-Thomson (1960) and Kochin, Kibel & Roze (1964). This formulation is based on the following assumptions, outlined by Milne-Thomson.

(i) The wake is composed of point vortices.

(ii) The complex potential will be *nearly* the same as for an infinite street of equally spaced vortices.

(iii) If the cylinder is surrounded by a control volume whose dimensions are large compared with those of the cylinder and with the street spacing and this contour advances with the velocity of the vortex street, then the motion on the boundaries will be steady.

(iv) The formation of the vortices is truly periodic.

While these assumptions at first glance may appear overly restrictive, they are satisfied to a reasonable degree in the wakes of vibrating bluff bodies. In the authors' previous work the vortices in wakes at a Reynolds number of 144 were corrected for viscous effects, but the assumption of point vortices is reasonable since a typical dimension of the contour which surrounds the cylinder is many times the viscous core radius of a vortex. The radius of the vortex cores was found by the authors to be $r_* \sim 0{\cdot}5$–$1{\cdot}0$ diameters in the wake of a vibrating cylinder. The use of the complex potential for an infinite vortex street neglects the presence of the body, but yields good agreement with the velocity field of the wake in the region where the street is fully developed with little lateral motion. Finally, the periodic street of vortices is an excellent approximation to the wake of a vibrating bluff body because of the increased two-dimensionality and periodicity in the wake as a result of body vibrations within the regime of synchronization.

The cylinder, vortex street and the surrounding contour are illustrated in figure 1. In co-ordinates fixed to the equilibrium position of the cylinder, the undisturbed flow velocity is U and the vortices move downstream with velocity fl, where f is the vibration (and vortex shedding) frequency and l is the longitudinal vortex spacing. If the contour $ABCD$ is fixed relative to the vortex street and all motions are viewed from that reference frame, then the cylinder, of diameter d, moves to the left with velocity $-fl$, the vortex street is stationary and the fluid far from the wake and ahead of the cylinder has velocity $U-fl$. This system is dynamically equivalent to the case of uniform flow past a cylinder. When an otherwise stationary cylinder is vibrating in the y direction, the increased vortex drag on the body should be evidenced by changes in the momentum and force balances for fluid contained within the contour $ABCD$. The steady drag is obtained in the classical development by integrating the instantaneous drag over

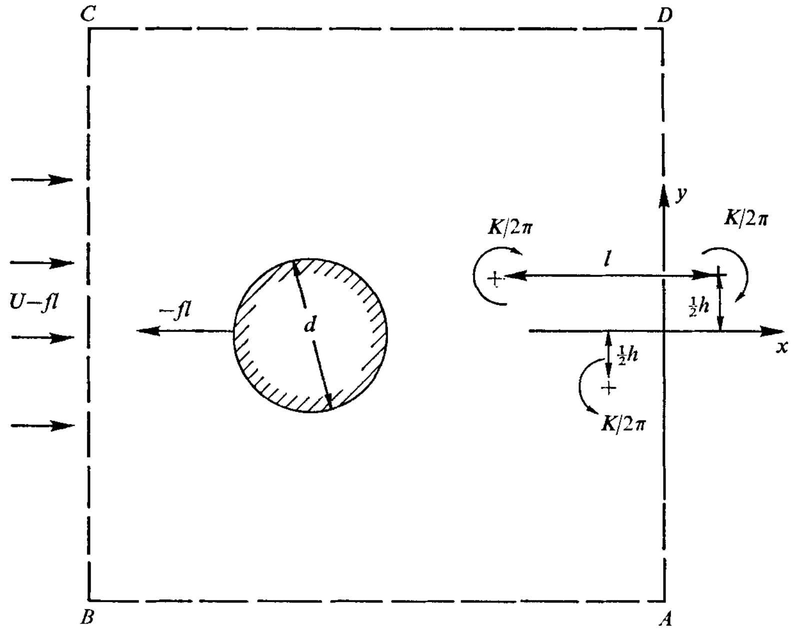

FIGURE 1. Schematic diagram of the contour $ABCD$ which bounds a control volume containing the cylinder and the vortex street generated by the cylinder. The contour is fixed relative to the vortices.

a period of the natural vortex motion; in the case of a vibrating cylinder, the period of the integration is equal to the synchronized periods of the vortex-street and the body motion.

The resulting vortex drag equation, following the developments of Milne-Thomson (1960) and Kochin *et al.* (1964), is

$$D = \rho K^2/2\pi l + \rho K(2fl - U)\,h/l, \tag{1}$$

where D is the drag force on the cylinder per unit distance along the body span, K is the initial circulation of the vortices and h is the lateral spacing of the vortex street. The von Kármán drag coefficient C_{DK} is then

$$C_{DK} = \frac{2D}{\rho U^2 d} = \pi\left(\frac{K}{\pi U d}\right)\left[\left(\frac{K}{\pi U d}\right)\frac{d}{l} + 2\left(\frac{2fl}{U} - 1\right)\frac{h}{l}\right]. \tag{2}$$

It is interesting to note that the same equation was obtained by Synge (1927) in his computation of the steady vortex drag on a bluff body. Synge, however, obtained his version of (2) after assuming that the fully developed vortex street was made up of a *semi-infinite* street of point vortices with constant lateral and longitudinal spacings. While the original development of (2) is independent of body shape, experimental results for the unknown parameters such as the vortex strength and spacings ultimately depend on the shape of the body which generates the vortex street. The results obtained with the von Kármán drag model are discussed in the next section, for the particular case of a vibrating circular cylinder.

St^*	a/d	f/f_s	l/d	h/l	$2(fl/U)-1$	$K/\pi Ud$	C_{DK}	$C_{DK}/C_{DK,0}$
Vibrating cylinder, Reynolds number = 144								
0·178‡	0·0	1·0	5·4	0·180	0·9	0·81	1·21	1·00
0·199	0·12	1·0	5·4	0·156	—	1·11	1·70	1·40
0·208	0·30	0·9	5·9	0·126	—	1·27	1·76	1·46
0·231	0·30	1·0	5·4	0·134	—	1·23	1·81	1·50
0·254	0·30	1·1	4·9	0·146	—	1·33	2·23	1·84
0·263	0·48	1·0	5·4	0·112	—	1·35	1·92	1·58
Vibrating cable, Reynolds number = 450								
0·208‡	0·0	1·0	5·0	0·180	0·88	0·64	0·90	1·00
0·270†	0·30	1·0	5·0	0·140	—	1·04	1·49	1·66

† Measurements made downstream from the cable antinode.
‡ Stationary-cable and stationary-cylinder reference values.

TABLE 1. Vortex drag on stationary and vibrating bluff bodies

3. Vortex drag on a vibrating bluff body

3.1. *Vortex strength and drag*

The von Kármán drag formula (2) has been used to determine the drag coefficients of vibrating bluff bodies, and the results are listed in table 1. All values of the vortex strength and spacing were obtained from a match with experimental results and no *a priori* assumptions were made in the determinations except that the vortex strength $K/2\pi$, the spacing ratio h/l and the viscous core radius r_* could be specified by satisfying a minimum mean-square-error criterion as discussed by Griffin & Ramberg (1974). In addition to the vortex strength and drag for a cylinder at a Reynolds number Re of 144, results from a study of the wake behind a vibrating cable at $Re = 450$ are included. The data for the cable result from measurements made downstream of the cable antinode, for vibrations at the Strouhal frequency f_s. As in the case of the vibrating cylinder, the longitudinal vortex spacings l for the stationary cable and cylinder respectively were measured directly and the values of the vortex strength $K/2\pi$, the spacing ratio h/l and the vortex core radius r_* were determined on the basis of the mean-square-error criterion.

An especially interesting result to be found in table 1 is the relation between the lateral vortex spacing h and the drag coefficient C_{DK} as the amplitude of vibration is increased at the Strouhal frequency. An increase in drag is usually thought to be accompanied by an increase in the lateral vortex spacing or the width of the wake, but the results for the vibrating-cylinder wake are opposite to this notion. The lateral spacing was found to decrease with increasing amplitude while the drag was simultaneously increased. (It had previously been shown that the longitudinal vortex spacing was independent of changes in vibration amplitude at a fixed frequency.) This change in spacing was obtained not only from hot-wire measurements but also from the flow-visualization experiments reported in Griffin & Ramberg (1974).

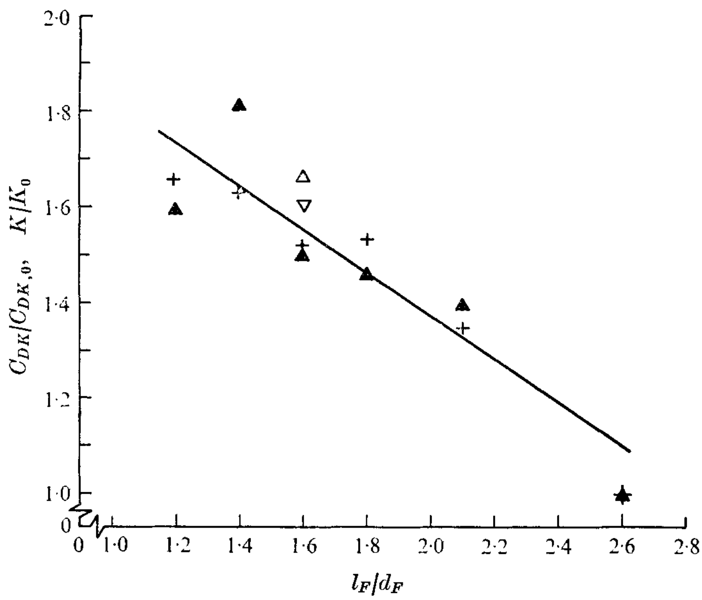

FIGURE 2. The vortex-street drag coefficient C_{DK} and the vortex circulation K as functions of the ratio l_F/d_F of the length l_F of the formation region and the wake width d_F at formation. Stationary-cylinder reference values: Reynolds number $Re = 144$, $St = 0{\cdot}178$, $C_{DK,0} = 1{\cdot}21$, $K_0 = 0{\cdot}81$; $Re = 450$, $St = 0{\cdot}208$, $C_{DK,0} = 0{\cdot}90$, $K_0 = 0{\cdot}64$. $+$, K, $Re = 144$; ∇, K, $Re = 450$; ▲, C_{DK}, $Re = 144$; $\triangle$, C_{DK}, $Re = 450$.

The drag coefficients $C_{DK,0}$ of the stationary cable and cylinder were also obtained using the von Kármán drag formula. The values of the stationary-cylinder vortex strength at $Re = 144$ and 450 were obtained from the results of Berger (1964) and Chen (1972), respectively, and the values of spacing ratio h/l were determined by extrapolating a plot of the authors' results at the Strouhal frequency to zero vibration amplitude. The results for the stationary-body drag coefficients are in good agreement with direct measurements at the respective Reynolds numbers (see Morkovin 1964). There is an inverse relation between the vortex strength in the wake of a vibrating cylinder and the length l_F of the vortex formation region (l_F is the initial downstream position of a fully formed vortex in the wake). All the results for the drag coefficient C_{DK} in table 1 are also plotted in figure 2 as a function of the length of the formation region. The inverse relation between the drag and the length of the formation region is evident, both for the vibrating-cylinder results at $Re = 144$ and for the vibrating-cable experiments at $Re = 450$. These results serve to demonstrate further the close relation between changes in the near wake flow of a bluff body (as characterized by the length of the formation region) and changes in the fluid-dynamic forces which act on that body. The results also support the supposition by many investigators that the increased fluid forces which act on a vibrating body directly correspond to increased vortex strength in the wake.

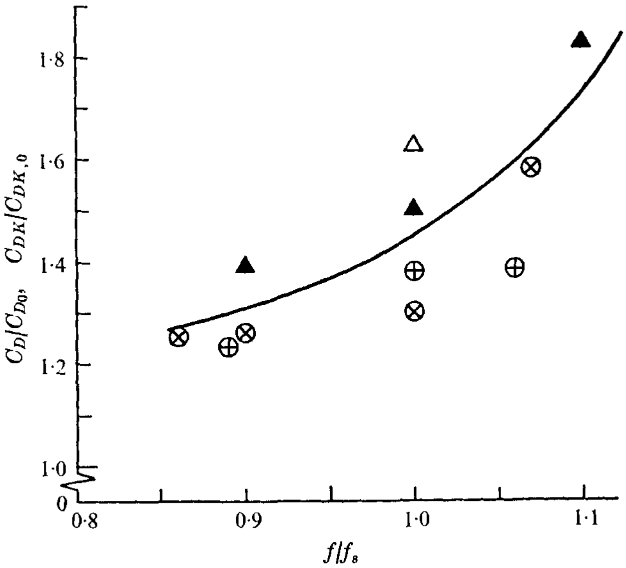

FIGURE 3. The drag coefficient of a vibrating circular cylinder as a function of the frequency f (normalized by the Strouhal frequency f_s) within the regime of wake capture or lock-in. Direct measurements of C_D (Tanida *et al.* 1973): ⊗, $Re = 80$, $St = 0{\cdot}14$, $C_{D0} = 1{\cdot}30$; ⊕, $Re = 4000$, $St = 0{\cdot}18$, $C_{D0} = 0{\cdot}90$. For symbols for von Kármán vortex-street drag C_{DK} (present study) see caption to figure 2. Cylinder amplitude, peak-to-peak, is 0·3 diameter (both cases).

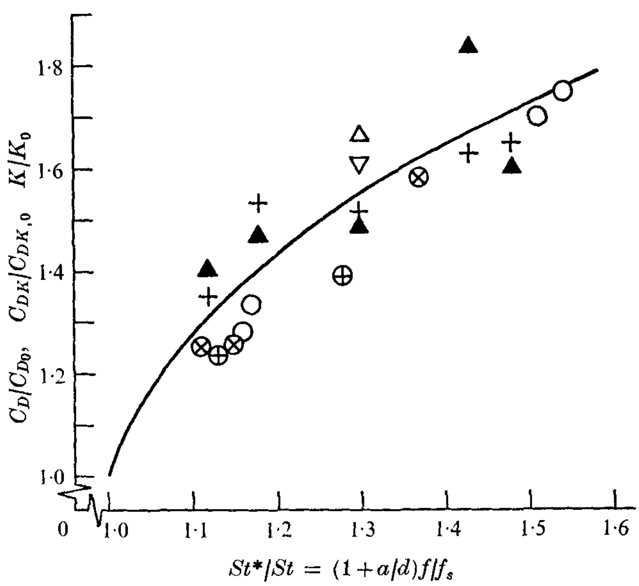

FIGURE 4. The von Kármán drag coefficient C_{DK}, the vortex circulation K and the drag coefficient from direct measurements of C_D as functions of the wake-capture Strouhal number St^* [see (3)]. For symbols for von Kármán drag coefficient C_{DK} and vortex circulation K see caption to figure 2. ○, drag coefficient C_D, direct measurements, Griffin *et al.* (1973), $Re = 550$, $St = 0{\cdot}208$, $C_{D0} = 0{\cdot}91$. For symbols for direct measurements of Tanida *et al.* (1973) see caption to figure 3.

3.2. *A comparison with direct force measurements*

It is useful to obtain a further measure of the validity of the von Kármán drag behaviour by means of a comparison with direct measurements of the drag on vibrating circular cylinders. Tanida *et al.* (1973) have measured the drag on a vibrating cylinder for a wide range of experimental conditions at Reynolds numbers of 80 and 4000. In their experiments a cylinder was towed through still water in a tank and vibrated at frequencies both inside and outside the lock-in regime and both transverse and parallel to the tow direction. The results from the previous section are compared with some measurements of Tanida *et al.* in figure 3. The drag coefficients C_D of the vibrating cylinder were normalized by the stationary-cylinder values in order to minimize any Reynolds number effects and to obtain a meaningful measure of the drag amplification at various frequencies within the lock-in regime. Fortuituously, the amplitudes of vibration for the two sets of experimental data are very nearly equal, which allows a straightforward comparison. This figure shows fair quantitative agreement between the direct force measurements and the steady drag computed from wake measurements of vortex strength and spacing. The drag amplification that accompanies the vibrations is clearly shown in both cases.

Further evidence of this behaviour is given in figure 4, where the von Kármán drag coefficients C_{DK} from the present paper are plotted with direct measurements of C_D from Griffin *et al.* (1973) as a function of the parameter St^*. This parameter is related to the usual Strouhal number St by

$$St^* = St\left(1+\frac{a}{d}\right)\frac{f}{f_s}, \quad St = \frac{f_s d}{U}. \tag{3}$$

St^* is a wake-capture Strouhal number based on the vibration frequency f, the length scale $a+d$, where a is the peak-to-peak amplitude of the motion, and the incident flow velocity U. Again there is a correspondence between direct force measurements and the drag coefficients which result from a wake velocity survey. The additional direct measurements in the figure were obtained using an elastically mounted rigid cylinder which underwent vortex-excited resonant oscillations in a wind tunnel at Reynolds numbers between 550 and 900. The data for the vortex strength and vortex-street drag and the direct drag measurements generally fall onto a single curve when the differences in the Strouhal numbers of the several experiments are taken into account. The Strouhal numbers for the stationary cylinders and cables in both air and water ranged from 0·18 to 0·21 for the experiments performed at NRL and from 0·14 to 0·18 for the experiments of Tanida *et al.* The results plotted in figure 4 further confirm the correspondence between the wake response, as characterized by the vortex strength and length of the formation region, and the fluid force amplification, as characterized by the steady drag, when a bluff body is vibrating under conditions of wake capture or lock-in.

4. Summary and conclusions

The von Kármán drag formulation has been employed to determine the drag on vibrating circular cylinders. The cylinders were forced to vibrate in a plane normal to an incident uniform stream, and at all times the frequency of the vibrations was locked to the frequency of vortex shedding.

An inverse relation has been obtained between the steady drag coefficient of the vibrating cylinder and the length of the vortex formation region behind the cylinder. This result further confirms the close relation, previously shown by the authors, between changes in the flow-induced forces and changes in the wake flow which accompany the vibrations. The von Kármán drag coefficient was increased by the vibrations by as much as 60–80 % from the value obtained when the cylinder was stationary.

The von Kármán drag coefficients for a vibrating cylinder and cable, obtained from wake measurements of vortex strength and spacing, were found to be in agreement with direct force measurements on vibrating cylinders. These direct measurements were made at Reynolds numbers of 80, 500–900 and 4000 while the wake measurements were made at Reynolds numbers of 144 and 450.

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